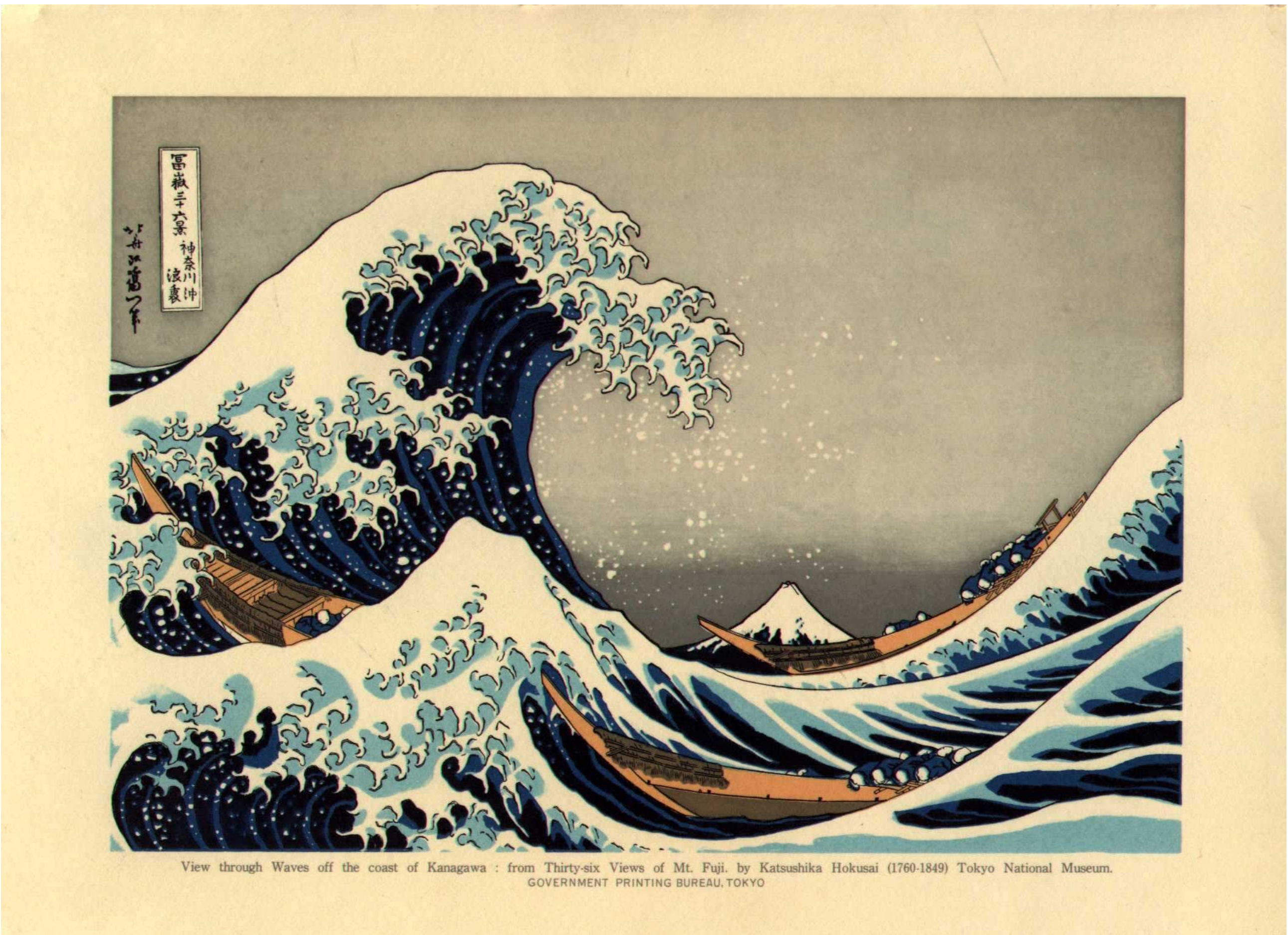
The Journey of Chaos



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THE JOURNEY OF CHAOS

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"The great book of nature is written in mathematical language"

– Galileo

Cover: View through waves off the coast of Kanagawa : from Thirty-six Views of Mt. Fuji. by Katsushika Hokusai (1760-1849) Tokyo National Museum.

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1 Introduction

This century has seen three major scientific revolutions – the theory of relativity, quantum theory and chaos theory.

> *"Relativity eliminated the Newtonian illusion of absolute space and time; quantum theory eliminated the Newtonian dream of a controllable measurement process; chaos eliminates the Laplacian fantasy of deterministic predicability."* [1]

Although chaos has only recently become a catch phrase in science, many of the world's oldest cultures have notions of chaos richly embedded in their mythologies and cosmogonies[i]. Interesting parallels can be found between many of the concepts of these ancient cultures, some of which can even be seen to tie in with the discoveries of modern chaos theory.

With the arising of modern scientific principles in Greece, the world went through a period in which chaos was all but forgotten, being considered an invalid concept. The natural world was to be fundamentally ruled by orderly systems, although sometimes the order was too complex to be understood. Building on this concept of order, many scientists envisaged the power to predict the future of the universe forever. At the time, it was an absolutely mind-blowing concept.

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[i] Cosmogony = theory of the origin of the universe.

The Greek perception of an ordered universe held firm for over two thousand years until developments in technology mid-way through this century. Chaos theory arose out of a newfound facility to harness the power of computers for mathematical and physical analysis.

Chaos theory uncovered what became known as ‘deterministic unpredictability’ – a chaotic situation arising from an equation or system that has no outside influences or hidden secrets. It became apparent that incredible complexity could arise out of very simple equations. In fact, order and chaos were found to closely linked, the one often arising out of the other. As science has matured, it has realised how much it cannot predict.

Chaos theory draws together a wide range of sciences, including ecology, economics, meteorology, geometry, mathematics, physics, electronics and astrophysics. It finds common themes between previously unrelated scientific disciplines.

2 Chaos in Mythology

Many of the world's oldest cultures have references to chaos in their mythology and creation stories. The Chinese, Egyptians, Mesopotamians, Babylonians, Indians and Greeks are amongst them. Common to them all is the notion that order arises out of an infinite state of chaos to form the cosmos or ordered universe. This process either occurs of its own accord or is assisted by a god or god-like figure.

Cosmogonical chaos has many different names. Sometimes it is directly referred to as chaos itself, exemplified by the Egyptians and the Greeks. It is also referred to as a metaphor: Hun-tun in Chinese mythology, Asat for the Vedic Indians, and Tiãmat in Babylonian cosmogony.

2.1 Chinese Mythology

The Chinese have direct references to chaos in their stories of creation. One mythological Chinese emperor (analogous to the gods of the Greeks and the Romans) had the name Hun-tun, meaning chaos.

> *"…for the Chinese as for other people, creation was the act of reducing chaos to order."* [2]

The writings of Chuang-tzu[ii] relate the story of the end of chaos and the beginning of the world. In this myth, the emperor god of the Northern Sea, Hu, and the emperor of

[ii] Chuang-tzu was a Taoist writer in the third century BC.

the Southern Sea, Shu, used to meet, on occasion, halfway between their respective confines in the territory of Hun-tun, the emperor god of the Centre. Hun-tun, although being most welcoming, differed from the others in that he did not possess the seven orifices (that is, mouth, ears, nose, and eyes).

Hu and Shu were most grateful for Hun-tun's hospitality, so they resolved to bore the orifices required for sight, hearing, eating, and breathing into Hun-tun. This they did, boring one hole a day. On the seventh day, as the final orifice was completed, Hun-tun died. With the death of chaos, the world came into being. The Chinese word for lightning is shu-hu, which is a combination of the names of the North and South emperors, and hence, a stroke of lightning may also have been involved.[iii]

Another Chinese text from the third century AD describes chaos as a hen's egg. This egg is depicted as the yin-yang symbol popular today (☯).

> *"The separation of chaos into an initial yin and yang is to be found as a fundamental concept in Chinese thought…"* [3]

Phan-ku, who is usually depicted as a dwarf dressed in bearskin or leaves, hatched from this egg, and from it the heavens and the earth were created. The heavy elements formed the earth, or yin, while its pure elements formed the sky, or yang. For eighteen thousand years the earth and the sky separated at a rate of ten feet a day. Phan-ku grew at the same rate also, his body filling the space between. When Phan-ku died his body formed the earth's elements.

The concept of the world egg is not confined to China. Similar themes can also be seen in classical Indian cosmogonies, in which a world egg opens to form the heavens from its upper part and the earth from its lower.

Chinese mythology also has a story in which chaos re-emerges: the world of mirrors and the world of humans were not always separated. Although being quite different in form, the mirror beings and human beings lived together in harmony. One night, however, the mirror people invaded without warning. Their might was great, and chaos supervened. Human beings quickly realised that the mirror people were, in fact, chaos.

Only with the strength of the Yellow Emperor god did human beings manage to defeat the mirror people. In order to prevent any further uprising, the Yellow Emperor cast a spell that bound the mirror people to mimic the actions of men. However, the Emperor's spell was not strong enough to bind them to this task for eternity. One day, the spell would diminish and chaos would once again show its face.

2.2 Egyptian Mythology

Egyptian cosmogony, as with that of the Chinese, refers directly to chaos, although it takes a different form. For the Egyptians, chaos was an ocean, the predecessor of all else; boundless, it had existed for eternity. Although chaos was often described by the Egyptians as unexplainable and formless, it was never perceived to be immaterial.

[iii] One current theory that attempts to explain the origins of life on earth claims that life began from lightning striking the ocean, creating urea, which is a by-product of life, and glutamic acid, which is a protein precursor.

Creation was the act of giving 'watery chaos' definition and differentiation, not achieved by an act of a god, but by a force called the *demiurge*.

> *"Egyptians did not believe that the world had been created out of nothing: material of some kind had been there always. They imagined the original creation as a shaping of that formless material into an ordered world… The world had not been shaped by a god who had existed for ever and ever – what had existed for ever and ever was chaos. Often chaos is described in negative terms: it cannot be explained…Yet chaos was not imagined as immaterial: it was a boundless ocean, called Nun."* [4]

The Egyptians viewed their enemies as the forces of chaos. If Egypt was conquered and ruled by an enemy king who chose not to worship the Egyptian gods and become Pharaoh, it was seen as a victory for chaos. They saw themselves and their kings as fighting against the "agents of chaos," in all situations, in the name of the cosmos. Indeed:

> *"Every Near Eastern world-view showed an awareness not only of order in the world but of the instability of that order."* [5]

The Egyptian cosmogony of the Hermopolitan priests also refers to a cosmic egg that hatched upon the primeval mound. From it was born the sun-god, who brought order to the world. According to this tradition, chaos had four characteristics, represented by the gods and goddesses of primordial water, infinite space, darkness, and invisibility. Together, these gods were known as the Hermopolitan Ogdoad. According to the legend of Khmunu, the egg was laid by Thoth, the god of wisdom.

2.3 Dragons in chaos myths

Another link between Chinese and Egyptian chaos mythology is the use of dragons, although their views on their significance differ. While the Egyptians saw the dragon Apophis as representing chaos, the Chinese saw dragons as embodying the principle of order (yang).[6]

The myth of Ra, the sun god, taken from Egyptian Mortuary Texts, describes the battle between the God Ra and Apophis. Ra was victorious, overcoming the forces of darkness and disorder, but the dragon is not mortally wounded.

It is significant that Ra does not succeed in killing the dragon, only suppressing him. This harkens back to the Myth of the Yellow Emperor (see section 2.1), where chaos is only laid dormant and not eradicated.

2.4 Vedic Indian Mythology

Vedic Indian mythology also has a linkage to that of the Egyptians and the Chinese. It involves the story of the warrior god Indra. The section of the story that is of interest begins before the ordered world had come into being, during a period where the *Adityas* and the *rakshashas* are at war. The Adityas are generally portrayed as being human in form although possessing superhuman powers, while the rakshashas are demons, usually taking the form of serpents, dragons, or sometimes boars.

The Adityas are descended from the goddess Aditi, whose name means freedom, and standing for freedom, they want the cosmic waters to be released. The demons, whose names often mean restraint or bondage, contain the cosmic waters, which are guarded by Vritra, the arch-demon.

The Adityas call upon the help of Indra; perhaps they even instigate his creation. The youngest of the gods and eager to make a name for himself, Indra was born of the sky (his father) and the earth (his mother). To prepare himself for battle, Indra took three draughts of the stimulant soma. This caused him to grow so large as to fill the earth and sky. His parents leapt apart in shock, never to be reunited.

Indra fought Vritra armed with the thunderbolt, *vajra*, and after a furious battle, the arch-demon lay dead. Through this victory, Indra became king of all the gods, and as the cosmic waters flowed from the belly of Vritra[iv], as order flowed from chaos, he set about making the ordered world.

> *"Vritra is very much like the Egyptian Apep or Apophis. Like him, he represents primordial chaos: dwelling in everlasting darkness, he contains the cosmic waters. By attacking, piercing and slaying him, the warrior god – Indra…– sets those waters free."* [7]

[iv] Note: accounts vary as to whether the cosmic water flowed from a cave guarded by rakshashas, or whether they flowed from Vritra's belly.

Indra's use of a thunderbolt in defeating Vritra harkens back to the Chinese story of Shu and Hu, where lightning is involved in the emergence of order. Indra has another link with Chinese mythology, through the story of Phan-ku. Both Indra and Phan-ku are born of the earth and sky, and grow to fill the space between.

2.5 Mesopotamian & Babylonian Mythology

For the Mesopotamians, a culture dating back as far as 3,000 BC, the world began with a boundless ocean, akin to that mentioned in Egyptian mythology.

> *"…in the earliest Mesopotamian world-view, there was nothing but salt ocean, primordial, boundless."* [8]

From this developed the earth and the sky, although they were still joined to the ocean. A god, creating the present form of the world, separated them.

> *"It was his [An's] will that lifted existence out of chaos and established the world order."* [9]

An is the God of the Sky, and he is the father of all things. However, he was not the first in existence. Out of the boundless ocean of chaos was born the sky, the god of which set out to create the ordered world.

The Babylonians had several gods representing different manifestations of chaos. One of these was Tiãmat, a primordial chaos goddess of salt ocean.

"She and other early gods embodied the various faces of chaos. For example, there was a god symbolising the boundless stretches of primordial formlessness, and a god called 'the hidden,' representing the intangibility and imperceptibility that lurks in chaotic confusion." [10]

In this way, the Babylonians acknowledged a type of order within chaos. By naming *forms* of disorder, a form of implicit order is already being conceived.

2.6 Greek Mythology

The Greek mythology, like that of the Egyptians and Chinese, has chaos embedded in its stories of creation, although the timing of its inception varies. One source, *Arogonautica Orphica 12*, states that *chronos*, or time, was the first to exist. Another, that of Plato, indicates that Uranus and Gaia were the primordial beings.

The majority of sources, however, identify chaos as one of the earliest cosmogonical entities. Hyginus states that chaos arose out of primordial mists. According to the *Hesiod*, chaos was the first into existence, and from it came Nyx (night), Gaia (earth), Erebus (pure darkness of the underworld), Tartus 1 (the lowest abyss beneath the earth), and Eros (the god of love). It is up to interpretation as to whether these entities are the offspring of chaos or whether they just came into being of their own accord. In the cosmogony of Ovidius, first of all "*was what man called chaos: a rough unordered mass of things."* [11] God then gave the world order. Aristophanes, a Greek playwright who lived around 400 BC, also saw chaos as the first to exist, although not alone, since Nyx, Erebus, and Tartus 1 also accompanied chaos in the beginning.

As with the Chinese, the Indians, and the Egyptians, Aristophanes' account involves a cosmic egg. Nyx laid an egg in Erebus, from which hatched Eros, who "caused all things to mingle." This reference to Eros and the egg probably implies the beginning of life. Love can also be viewed as reproduction, as suggested by the word 'mingle', with the egg being a symbol of fertility and life.

2.7 Water in chaos myths

There are a variety of cultures that have water as the initial cosmogonical entity. As well as those already mentioned, these include the Maidu Indians of California, the people of the Marshall Islands, Russian Altaic, the Hurons (American Indians), and to a lesser extent the Yoruba (of Nigeria).

The themes of chaos and that of the primordial, boundless ocean are recurrent throughout many cosmogonies. The Egyptians and the Mesopotamians provide a key linkage between the two, giving chaos the form of an ocean. Clearly for both, chaos is a force that must be in some way controlled.

Oceans would provide an accurate metaphor of chaos for many cultures attempting to describe that which the Egyptians viewed as unexplainable. Oceans are vast, seemingly infinite. Some cultures, notably the Vikings, believed that the ocean continued to the end of the world and disappeared over the edge. It was an infinite source that continued beyond the horizon of human perception.

More modern research into fluid dynamics has discovered an infinity of a different kind. One of the earliest researchers in this field was Leonardo Da Vinci (see section 5.4.1). He found that eddies break up into eddies of decreasing size, a form of infinite complexity, a recurring theme in today's scientific view of chaos.

Oceans are also turbulent and unpredictable, two aspects which are key to the modern understanding of chaos. From the earliest research into the movement of water, it has been found to be turbulent when there is large flow. The work of Leonardo da Vinci also represented this. In this way the ocean represented something of infinite size and infinite complexity.

Oceans are also a prime example of that which is the same on varying scales and quantities. A wave breaking on a sandy riverbank takes the same form as a six-foot wave breaking at a top surf location. This is akin to the self-similarity of fractals, where small sections show a likeness to the overall image.

When endeavouring to relate chaos as an experience tangible to the everyday world, the ocean is likely, for many cultures, to have provided the most accurate metaphor.

3 The Forgetting of Chaos through scientific reductionism.

One of the difficulties encountered when trying to comprehend infinity is that there are no reference points. Imagine a boat on an ocean. It is night and the sky is clouded over. As far as can be seen there is nothing but water. Without reference points, it is impossible to gain any understanding of where we are. Therefore, humans add reference points, or attempt to create order out of the ocean of chaos. Through reductionism, science has taken this to the extreme, and in so doing, the mythological link humankind had with chaos has become nullified.

> *"The psychologist, anthropologist, and critic René Girard has observed that we humans have a great need to interpret the disorder in myths from the point of view of order. 'Even the word 'dis-order' suggests the precedence and pre-eminence of order,' he says. 'We are always improving on mythology in the sense that we suppress its disorder more and more.' "* [12]

The same holds true in science, and with the birth of the modern scientific method in Greece, chaos began to be forgotten.

3.1 The Birth of Modern Science in Greece.

Despite strong ideas of chaos in their mythology, the Greeks were among the first to attempt to understand their surroundings through reductionism. In other words, they broke things down into their smallest possible components, tried to understand them individually, and then added them up again in order to comprehend the whole.

> *"Rationalism, for whatever its value, appears to have emerged from mythology with the Greeks… There was a feeling that the natural laws, when found, would be comprehensible. This Greek optimism has never entirely left the human race."* [13]

Many of the Greek areas of study reflect their desire to understand the world by its smallest and most simple parts. For example, there was much discussion regarding atoms and indivisibles, a concept that continues to be relevant today.

> *"…that most influential concept of early Greek science, the atom. The notion of atoms was offered as the bedrock of understanding, and its properties seemed to symbolise the supposedly ultimate form of reasonable questions that could be raised about the universe."* [14]

Through reductionism, the Greeks were also the first to encounter some of the problems it raised. On the mathematical frontier, several of these were later solved by calculus. This process of reductionism began with the Greek philosophers, Thales, Anaximander, and Anaxagoras who took the mythological idea of chaos as a creative force and applied it to science.

"[They]… proposed that a specific substance or energy – water or air – had been in chaotic flux and from that substance the various forms in the universe had congealed." [15]

Thales, born in Miletus in 624 BC is credited by later Greeks as being the founder of Greek science, mathematics, and philosophy. Thales' mother is believed to have been a Phoenician and this may be one of the reasons why he received an Eastern education. Although many details about his life are sketchy, it is certain that he spent time in Egypt and most probably Babylonia and would have had a grounding in creation myths from these and other lands.

"It may be that what seemed to the Greeks a multiplicity of achievement was simply the lore of the more ancient peoples." [16]

Thales took the knowledge of the East and brought it to the West. However, in doing so, he made the important advance of turning such knowledge into abstract studies. He was the first to attempt to prove mathematical statements using a logical series of arguments. Importantly, he was also the first to ask the question 'of what is the Universe made?' from a purely scientific perspective without relying on mythological ideas.

In this way, Thales began changing previous notions of chaos and creation into more rational scientific ideas. He proposed water as the fundamental element of the universe, still showing parallels to the mythological idea of the universe being created from a primordial and boundless ocean.

An example of science taking over mythology can be seen in a discovery of Pythagoras regarding the morning star (then known as Phosphorus) and the evening star, known as Hesperus. He proved that they were, in fact, the same star, and consequently it was renamed Aphrodite (and subsequently Venus by the Romans).

In the Pythagorean approach, we can see the beginnings of a reductionist view of the world. They viewed the world through numbers and geometry.

> *"In their world view lines were derived from points or unit numbers, from lines surfaces, from surfaces simple bodies, from these the elements and the whole world."* [17]

> *"They also held truth, intelligibility, and certitude to be cognate to numbers, which they contrasted with the erroneous world of the undefined, uncounted, senseless, and irrational."* [18]

Aristotle moved further away from mythological ideas of chaos and order, proposing that order was pervasive. That which appears chaotic merely has a high complexity of order, too complex to be understood at present.

Nevertheless, some Greek philosophers proposed that eventually, the universe would revert to a state of disorder from which a new universe would arise.

3. 2 From the Renaissance to the Present: Reductionism to Chaos.

Scientists continued the Greek method of reductionism for centuries. If a system appeared chaotic and unpredictable, science attempted to reduce it with the belief that, at a fundamental level, the system was ordered, linear, and predictable. Driven by this idea, scientists developed theories that became minimalistic. Since the time of the Greeks, science had been working towards a complete understanding of the universe. Later, prominent scientists and philosophers like Galileo, Descartes, and Newton made this seem astonishingly close to being reality.

> *"Traditionally, scientists have looked for the simplest view of the world around us."*[19]

The mathematician, astronomer, and physicist, Galileo (1564 – 1642), studied pendulum motion with this objective in mind. He discovered that the time it takes for a pendulum to complete a cycle is always the same, regardless of the size of the swing. That is, the speed and the size of the swing are always proportionately identical. After producing this hypothesis, he tested it by asking his friends to count the swing over the course of several hours. Although this is not the soundest method of confirming a hypothesis, the theory is an elegant one, and became widely accepted.

Galileo is also famed for his observations on the then unexplained force of gravity. In his experiments, he found that weight does not effect the speed with which an object falls to the ground. Throughout his work, he was simplifying the laws that govern the earth.

During the seventeenth century, Newton developed his theory of gravity and, along with Leibniz, calculus. Through understanding these new, all encompassing laws of physics, it seemed that it would not be long before the entire workings of the world could be explained and calculated. During the Napoleonic era, the French physicist Pierre Laplace, excited by this possibility, envisaged a law that could explain every physical phenomenon in the universe. With approximate knowledge of the present, he stated, it would be possible to predict an equally approximate future.

Indeed, by 1980 it seemed that physics had come so far that the end of unpredictability was in sight. The cosmologist, Stephen Hawking, occupant of Newton's chair at Cambridge University, spoke for most of physics when he said during a lecture entitled 'Is the End in Sight for Theoretical Physics?':

> *"We already know the physical laws that govern everything we experience in everyday life… It is a tribute to how far we have come in theoretical physics that it now takes enormous machines and a great deal of money to perform an experiment whose results we cannot predict."* [20]

Despite Hawking's confidence in the power of modern physics to predict the future, many experiments considered simplistic by most physicists can, in fact, display unpredictable behaviour, as in the example of pendulums.

> *"Students for generations have regarded pendulums as classical examples of simple, regular motion. In fact, pendulums still hold great surprises in store for us."* [21]

Thanks, primarily, to the work of John Miles of the University of California, pendulums are now seen as a good example of 'deterministic chaos'. David Tritton, of the University of Newcastle upon Tyne, explains a relatively simple experiment that exemplifies this. It involves a ball suspended from a piece of string. The string is attached to a horizontally oscillating crankshaft. The crankshaft drives the motion of the pendulum.

When the crankshaft is driving the pendulum slightly higher than its natural (free-swinging) speed, the motion of the pendulum increases accordingly, before developing a secondary movement that runs perpendicular to the drive. This causes the pendulum to move in a circular path. Once the pendulum has settled into this path, it will continue as long as the oscillation of the crankshaft is maintained. Although this motion is predictable and non-chaotic, it does contain an unpredictable element: the initial direction of the pendulum (clockwise or anti-clockwise) is a random event. Once the direction becomes established, however, the course of the pendulum is easily predicted.

If the crankshaft is driving the pendulum slightly lower than its natural speed, there are many possible outcomes to the pendulum's movement, all of them elliptical with successive orbits never being identical. Over longer periods of time, significant change can be noticed. Not only is the motion aperiodic, it also frequently changes between clockwise and anti-clockwise. Experimental and theoretical work by Miles suggests that, indeed, there is no pattern to the movement. It is entirely chaotic.

4 The Originators of Chaos Theory

Scientists have always looked for theories and proofs that are pleasing and satisfying. It is a natural desire that stems from human aspirations to perfection. For example, according to calculus, adding 1+1/2+1/4+1/8…∞ = 2. This seems intuitively correct, and, as a consequence, science often sets out to prove such theories. In so doing, small errors are often encountered but ignored, being put down to inaccuracies in scientific method or measurement. For many centuries, mainstream science attempted to carry out experiments that excluded interferences such as friction. Friction and other non-linearities (see appendix 3) were considered a form of imperfection. Friction, however, is to be found everywhere in the world and known universe. The orbit of the moon is affected by friction created by the oceans, and simple everyday actions such as walking would not be possible without it. Eventually, science came to a point where non-linearity could not be ignored.

Logic tells us that adding up the individual parts should give us the whole. But what we also know from experience is that there are critical points where a small movement has a disproportionately large effect. Earthquakes offer a prime example. The slow but steady displacement of the earth's tectonic plates has been creating a tension between two surfaces. For many years nothing happens, friction prevents any offset movement between the surfaces. Then a critical point is reached. One plate moves an additional fraction of a millimetre and New Zealand experiences a severe earthquake that destroys hundreds of building and kills ten people (and fifty sheep!). Looking on a microscopic scale, it could be said that the tectonic plate moving a fraction of a millimetre caused the earthquake. Although this is true, it does not make sense without also considering the years of prior displacement. Chaos theory helps to explain this.

Unlike other scientific discoveries, where developments are individual achievements, chaos theory was discovered and explored by many scientists from varying backgrounds. Some of the earliest, like Poincaré, Julia, and Fatou, did not have available to them the technology to make their findings renowned, and receded into the background. Since the advent of the computer, however, the calculating power has been available to expose chaos with broad application.

4.1 Henri Poincaré

As far back as the late nineteenth century, a physicist, mathematician and philosopher from France called Henri Poincaré saw the possibilities of deterministic chaos existing inside closed Newtonian systems.[v] A pendulum swinging in a vacuum, not impacted by friction in any way, is seen as a closed system, that is, it has no outside influences. In Poincaré's era, any disorder in a system was seen as an outside influence that would disappear if it were possible to emulate a closed system environment. Poincaré's work was one of the earliest contradictions to these views.

Newton's laws of planetary motion are capable of predicting the orbits of two planets in a closed system environment, for example in a universe consisting only of the earth and the moon. Matters become slightly more problematical, however, if a third body

[v] A Newtonian system is a system that uses Newton's laws. These systems came before chaos and quantum theory and were thought to be predictable.

is added to the equation, for example, the sun. In fact, according to science writer, John Briggs and physicist, F. David Peat, Newton's equations become unsolvable.

In order to solve equations involving orbits of more than two bodies, a series of ever-smaller approximations are used to arrive at the answer. Poincaré contemplated what would happen if these approximations had an impact over a long period of time. Looking at the problem mathematically, it was non-linear, but nonetheless, it appeared that the introduction of the third body had little effect, for the most part. He did, however, discover that certain orbits caused a planet to wobble and then fly off course, even out of the solar system. This could have huge implications for our solar system, if over time, a series of planets happened to end up in one of Poincaré's chaotic orbits. Planets could suddenly start flying out of the solar system. Poincaré introduced one of the hallmarks of chaotic behaviour in an essay called "Science and Method" in 1903: 'sensitive dependence on initial conditions.'[22]

At the time, however, Poincaré's discovery was largely ignored as it was overshadowed by many of the other great scientific discoveries of the early twentieth century. Max Planck's work on quantum theory was challenging Newton's theories and Einstein was presenting his theory of relativity. Poincaré himself left his research, feeling overawed by his "bizarre" discovery.

Later, in 1954, three Russian scientists, A. N. Kolmogorov, Vladimir Arnold and J. Moser, collectively known as KAM, provided some of the answers to Poincaré's problem. Firstly, they noted that the condition Poincaré described could not occur if the third body had a gravitational pull less than that of a fly on the other side of the world. Secondly, they noted that it could only occur if the cycle of the planet's orbits fell in a ratio, that is, they repeated over a period of time. This means that the effect the third planet would have is one of positive feedback (see appendix 1), and, therefore, the change is amplified over time. If this is not the case, and the planet's orbits are quasi-periodic, then it demonstrates a form of negative feedback and is self-corrective.

What this positive feedback system shows is 'deterministic chaos'. This contradicted and nullified the idea that chaotic behaviour could not occur in a closed system.

Poincaré and KAM's theories are also backed up by evidence in our own solar system. Holes in the asteroid belt have been found where the latter coincides periodically with the orbit of Jupiter. Asteroids once in these zones have been sent flying randomly off into space. Some of the asteroids that have collided with the earth could be accounted for by such a theory.

According to Jack Wisdom of the Massachusetts Institute of Technology, many of the moons in our solar system must have undergone some periods of chaotic behaviour in the past but have since developed quasi-periodic orbits. One of Saturn's moons, Hyperion, appears to be undergoing one such period at the moment. Gaps in Saturn's rings are also possible results of KAM theory, although research in this topic is still under way.

4.2 Gaston Julia and Pierre Fatou

Throughout history, developments in geometry have run parallel to advances in other areas of science. The architecture and surveying of the Egyptians was only made possible by advances in geometry. The Greeks made many geometrical developments and applied them to practical science. For example, they were able to determine the distance of a boat from the shore using Pythagoras' theorem. They were also able to light fires using parabolic mirrors by focusing the sun's energy. In fact, according to Benoit B. Mandelbrot (the famed 'inventor' of fractals[vi]), Johannes Kepler's 17th Century discovery that orbits of planets could be described as ellipses was a catalyst for Newton's work on gravity. More recently, fractals have arisen as one of the leading geometrical fields. Crucial to their inception were Poincaré and the duo of Gaston Julia (1893 – 1978) and Pierre Fatou (1878 – 1929). They studied the dynamics of complex number maps around 1910, developing Julia Sets and laying the groundwork for modern fractal imagery.

Julia was an Algerian born mathematician. He had the misfortune of loosing his nose in World War I and carried out much of his mathematical research in hospital. Julia worked on iterative functions whereby the $f^n(z)$ stays bounded as n tends to infinity (where z is a complex number). Much less is known about the life of Fatou, although his work involved planetary motion. As with Julia, he had particular interest in rational functions with complex variables. Without the iterative power of computers, however, their research was limited and after a short period, the study was all but forgotten until the 1970s when Mandelbrot rekindled interest in the subject.

[vi] A fractal is an image possessing self-similarity (that is, the image repeats on different scales).

4.3 Benoit Mandelbrot

Benoit Mandelbrot was a Polish-born mathematician who, from the outset, had adopted an unconventional view. His educational background was also unconventional in that he claimed never to have learnt the alphabet or multiplication tables. He had a difficult childhood, fleeing from Nazi prosecution, due to his Jewish background.

In 1936, the family moved to Paris, where Benoit's uncle, Szolem Mandelbrot, lived. Szolem was a mathematician who was a founding member of Bourbaki. Bourbaki was a mathematical 'cult', designed to rebuild mathematics after World War I. Bourbaki's attitude was directly opposed to that of mathematicians like Poincaré who said, *"I know it must be right, so why should I prove it?"*[23] Bourbaki moved away from maths as a means of explaining physical phenomena, believing that maths was a science of its own and that it should not be judged by its application to other sciences. Indeed,

> *"A mathematician could take pride in saying that his work explained nothing in the world or in science…With self containment came clarity."* [24]

Bourbaki also rejected the use of pictures and geometry, believing them to be unreliable. Almost ironically, one of Mandelbrot's strengths was his ability to view things in pictorial form, allowing him to hide his lack of formal mathematical training at the prestigious École Polytechnique in France. At the time, however, this left him in the scientific wilderness. He ended up working at the Thomas J. Watson Research Center at IBM.

One of Mandelbrot's ideas that gained him most notoriety was the paper, "How long is the coast of Great Britain?" – a seemingly trivial question that yields the surprising answer – 'infinite'. When measuring the length of a coastline, the results depend on the detail of the measuring. If measuring a coastline from a satellite photo, one might choose to pick a point every hundred metres around the coastline and measure between these points. However, if greater accuracy is desired, a point every fifty metres may be measured. Although the coastline is still the same, the distance measured now proves to be greater, as more of the bays have been taken into account. The more detail in which we look at the coastline, the longer the coast becomes. Hence, at an infinite level of detail, the coast is infinitely long.

A simple geometric representation of this idea can be seen in the Koch Curve, in Mandelbrot's words, *"a rough but vigorous model of a coastline."* [25] The Koch curve is made up of equilateral triangles. An equilateral triangle one third of the size of its originator stems off each triangle. Therefore, the length of the perimeter of the Koch curve is 3 * 4/3 * 4/3 * 4/3…and ad infinitum.

Looking at the Koch curve from a distance, it appears that it is merely a twelve-sided star, or two superimposed equilateral triangles. Each 'edge' is as follows: moving closer in, the overall shape is maintained but the detail (and length of 'coastline') increases. If a section of it is magnified sufficiently, it appears identical to the original image.

This idea of self-similarity on differing scales is an idea central to fractals (see section 5.2.1).

4.4 Edward Lorenz

Ever since the early days of modern computers, meteorologists have seen them as an extremely useful tool. One such meteorologist was Edward Lorenz. He was born in America in 1917 and worked at the Massachusetts Institute of Technology. He had set up a computer (which he named ‘Royal McBee’) in his office that could simulate hypothetical weather for 12:00 am each day. It was a very simple model that worked on twelve variables, such as air pressure and wind speed. Each minute, a day would pass and the computer would produce a print-out telling Lorenz the weather. Despite its simplicity, it was a surprisingly realistic simulation, showing various patterns, although with a certain irregularity.

In 1961, Lorenz made the accidental discovery that small disturbances in initial conditions could result in unrecognisable results (see section 5.3). Through his chance finding, he was one of the first to stumble across deterministic chaos and fully realise its ramifications.

He termed this ‘the butterfly effect’; the small disturbance caused by a butterfly flapping its wings could be enough to create a storm on the other side of the world a few days later.

4.5 Robert May

Robert May was born in Sydney, Australia, and started his career as a physicist, before studying applied mathematics at Harvard. Following this, he developed an interest in biology at Princeton University in New Jersey. Because of his mathematical background, he brought a new approach to the study of population modelling, resulting in his investigation of bifurcations (see section 5.1.1). Using mathematics to model populations, May examined an equation as a whole, rather than looking at individual values. Previously, scientists had been trying to comprehend individual values in an attempt to find patterns and predictability. May was moving away from reductionism towards a holistic approach.

5 Fields of Application

"The theory of chaos touches all disciplines."[26]

Chaos theory can be applied to many diverse areas of science. These include fields such as biology, physics, chemistry, mathematics, electronics, and economics. In all areas it brought a new approach and often threatened old methods, and some cases encountered resistance from scientists with a more conservative outlook. Now that chaos has gained wider acceptance, however, it provides new insights and stimulates discussion between seemingly unrelated disciplines.

5. 1 Biology

Biology exemplifies chaotic process in many areas, from plant-like fractals to mathematical models of populations and predator-prey systems. Genetics, aspects of human biology, and many rhythmic behaviours of nature are also explainable through chaos theory.

5.1.1 Population Modelling

For ecologists and biologists, population studies frequently play an important role. Examining a population mathematically can be a useful tool, allowing current populations to be understood and future populations to be predicted. Mathematically, populations can be viewed as feedback loops; this year's population impacts directly on that of the following year. Finding an equation that matches a real life scenario is incredibly difficult – there are so many factors to take into consideration. A species can be affected by many predators, various food and habitat requirements (each of which depend on equally complex requirements), etc. It is not enough to know that birds eat moths, it is also necessary to know at what rate this occurs.

Since nature is highly complex, ecologists looked to simplified mathematical models as approximations. Firstly, these approximations measure population growth in regular intervals, not on a continual basic. The simplest of these imagines a world with infinite resources (food, habitat, etc) and no predators. The population increases like a compound interest sum, growing exponentially. Such an equation is as follows: $x_{next} = \lambda x$, where λ (lambda) is the rate of growth and x represents the population. If x takes on a value of 0.4 lambda takes on a value of 2 then:

$X_0 = 0.4$

$X_1 = 0.8$

$X_2 = 1.6$

$X_3 = 3.2$

$X_4 = 6.4$…

This equation, the Malthusian model, is a poor demonstration of the natural world, and its shortcomings are pretty obvious. The world is not a place of infinite resource, so the next task is to add a limit factor to the equation, representing the maximum size of the population. The following equation is still highly simplistic, but does solve the aforementioned problem: $x_{next} = \lambda x(1-x)$, where $x < 1$ and λ is the rate of growth. In this equation, the population is scaled between 0 and 1. (1-x) adds a limit to the population because as x rises, (1-x) falls accordingly. In this model, if x takes on a value of 0.4 and lambda a value of 2 then:

$X_0 = 0.4$

$X_1 = 0.48$

$X_2 = 0.4992$

$X_3 = 0.4999987$

$X_4 = 0.5$

$X_5 = 0.5$

The population has risen steeply to a state of equilibrium and settled to a constant value of 0.5. As the value of λ rises, the population behaves in similar fashion, but reaches a higher equilibrium. Here, λ takes the value 2.3:

$X_0 = 0.4$

$X_1 = 0.552$

$X_2 = 0.5687808$

$X_3 = 0.5641192$

$X_4 = 0.5655441$

$X_5 = 0.5651191$

$X_6 = 0.5652468$

$X_7 = 0.5652086$

$X_8 = 0.5652201$

$X_9 = 0.5652166$

$X_{10} = 0.5652176$

$X_{11} = 0.5652173$

$X_{12} = 0.5652174$

$X_{13} = 0.5652174$

$X_{14} = 0.5652174$

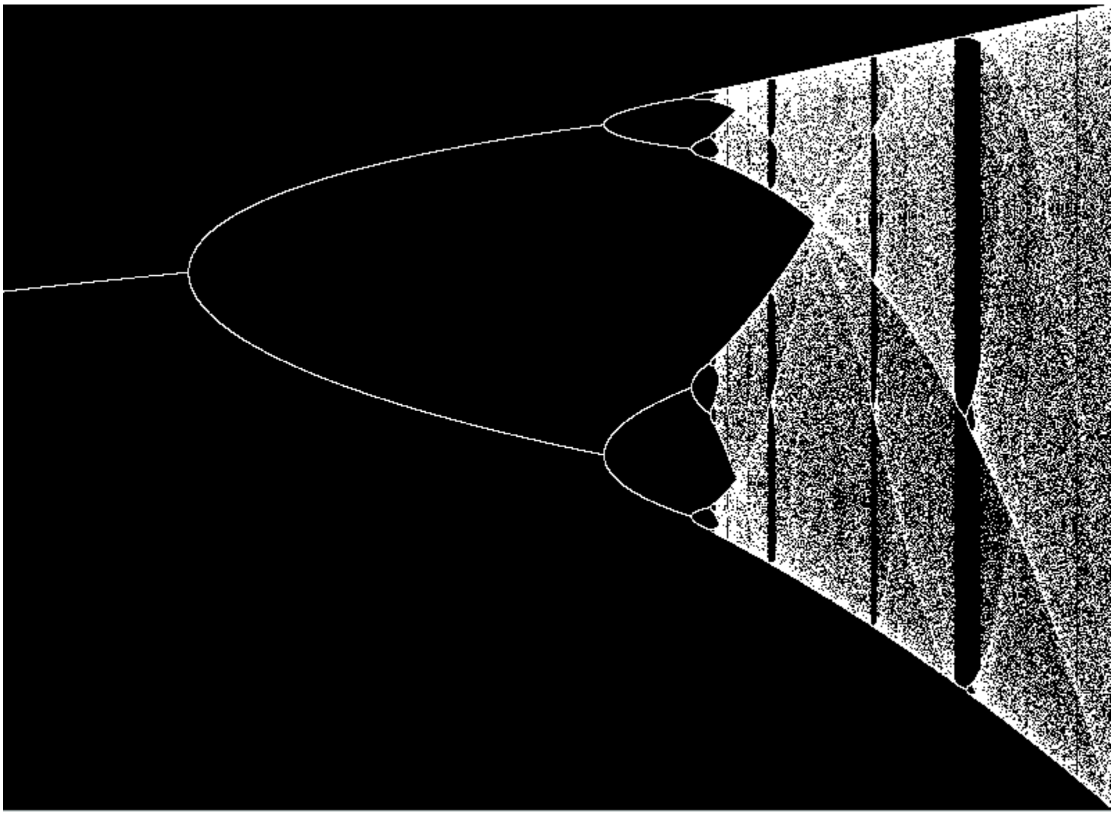
$X_{15} = 0.5652174$

Moreover, as lambda increases still further, the population no longer settles to a single number but begins to repeat in a series of two. Lambda = $1 + 5^{1/2}$ (= 3.236067977…):

$X_0 = 0.4$	$X_8 = 0.5000874$
$X_1 = 0.7766563$	$X_9 = 0.809017$
$X_2 = 0.5613325$	$X_{10} = 0.5$
$X_3 = 0.7968440$	$X_{11} = 0.809017$
$X_4 = 0.5238665$	$X_{12} = 0.5$
$X_5 = 0.8071737$	$X_{13} = 0.809017$
$X_6 = 0.5036755$	$X_{14} = 0.5$
$X_7 = 0.8089733$	$X_{15} = 0.809017$

Increasing lambda further, the pattern becomes more complicated. Rather than a series of two, the x repeats in a series of four, then eight and so forth. This process of periodic doubling is known as a *bifurcation.* Bifurcations were first studied by Robert May (see section 4.5). Before May, people had been looking at each iterative equation separately and looking for patterns and predicability within it. Robert May took a different approach. He took the last repeating values of a particular lambda and graphed these points next to that of the previous lambda. This enabled him to view the entire equation, not just a small part of it.

This image shows May's approach when x=.4, lambda increments = .0001, iterations (see appendix 2) = 5000, and plotted points = 50:



The above diagram shows this process of bifurcation. When the whole equation $x_{next} = \lambda x(1-x)$ is displayed, the areas of order and chaos can be clearly seen. Denser regions can be seen running through the chaotic regions, these show areas with a high population probability. Although the population could end up anywhere within the white area, the denser areas represent a mild form of order.

Regions of chaos in the diagram are periodically interrupted by regions of order, where the disordered mass suddenly reduces into an odd number of single lines, before dissipating into chaos once more. If these ordered areas are shown in greater detail, they are exact replicas of the first bifurcation. This shows a form of order that can be seen on different scales rather than on the same scale. The order cannot be seen on scale of time, but it can be seen on a scale of magnification. Although this idea only became popular with the advent of fractals, it is not entirely new. Similar ideas have been explored by mathematicians like Helge von Koch, who investigated the Koch curve, showing an infinite distance in a finite line. The Sierpinski triangle is another similar example (see section 5.2.1).

When lambda approaches greater values still, all traces of order vanish and chaos ensues. Here lambda = 4.

$X_0 = 0.4$	$X_6 = 0.02546474$
$X_1 = 0.96$	$X_7 = 0.09927552$
$X_2 = 0.1536001$	$X_8 = 0.3576796$
$X_3 = 0.5200284$	$X_9 = 0.9189796$
$X_4 = 0.9983954$	$X_{10} = 0.2978244$
$X_5 = 0.00640793$	

This process shows the hallmarks of a chaotic system. With low input, the system is orderly, with increased input, the system becomes complex and eventually chaotic.

5.1.2 Chaos Theory as a Modelling Tool

As well as being used in population modelling, chaos theory has also been used to describe other areas of biology, from the fractal-like structure of blood vessels and capillaries to many rhythmic systems found within nature. Research under way at the University of Newcastle in Australia suggests that linked oscillators, themselves exponents of chaotic principles, provide excellent models for such behaviour. From insects like glow worms and cicadas to rhythmically contracting tissues in the gastro-intestinal system, living things display a broad range of rhythmic activity, for which oscillators working on chaotic principles may provide the best explanation. Longer rhythmic behaviour can be seen in the plant kingdom, with the opening and closing of leaves and flowers. On a longer scale still are seasonal variation, ovulation, and predator-prey population growth. Nature provides much rhythmic synchronicity.

However, disrupting these cycles can lead to chaotic behaviour before order re-emerges, as Mohammad S. Imtiaz, of the University of Newcastle, explains: "One example is the membrane voltage recording from a freshly dissected stomach tissue. Initially these tissues produce a very non-coherent output. But as time goes on most of them slowly develop a regular rhythmic pattern. A possible explanation is that the cells become de-coupled in a tissue that has been dissected, a kind of mechanical trauma. Slowly as the tissue starts recovering, the cells start to communicate with each other and a rhythmic activity emerges. We are still working on this and it is still a mystery to a large extent."

He goes on to give another example, involving insulin-secreting cells. When a few insulin-secreting cells are isolated they do not produce clean, rhythmic behaviour. However, as soon as a number of them are grouped together, a clean synchronised activity emerges.

Although the processes involved in this transition are still largely unclear, it appears that chaos of this form is a transitory behaviour.

5.1.3 Genetics

One of the puzzles of genetics is how a seed can contain all the information needed for the growth of a huge tree. However, research in fractals has shown that images of infinite complexity can be created using random process combined with a few very simple rules. The Sierpinski triangle (see section 5.2.1) is a good example. Similar rules can be used to generate more natural formations, and, as discovered by mathematician Michael Barnsley, of Georgia Institute of Technology, it is possible to find rules for objects in the natural world. Based on the principle that nature possesses self-similarity, Barnsley was able to create a rule that reproduced an exact replica of a black spleenwort fern that *"no biologist would have any trouble identifying."*[27] Although the method, known as the 'collage theorem,' is complicated, simple rules can be found for any of the many natural shapes that possess self-similarity.

Admittedly, evidence is inconclusive, but Barnsley has, at least, made plausible the idea that plants store genetic information in a similar way.

5. 2 Computer Imagery

The geometry of chaos theory requires computers, for the most part, to generate the highly complex images involved. Surprisingly, these images are generated using relatively simply iterative complex number equations.

Computer imagery has played a large part in chaos theory's appeal with the wider public, the colourful fractal images striking a cord with the layman's artistic sense.

5.2.1 Fractals

Julia Sets, some of the earliest fractals to be explored, were first experimented with and discovered by the duo of Gaston Julia and Pierre Fatou (see section 4.2). Julia Sets are maps on a complex number plane. A complex number z contains two parts, a real part and an imaginary part. That is, $z = x + iy$, where x and y are real numbers and i is the imaginary part, $-1^{1/2}$. When working with complex numbers, the real and imaginary parts are considered separately, as in normal algebra.

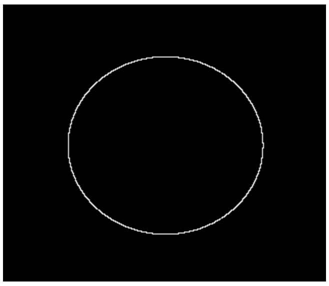
$$
\begin{aligned}
z^2 &= (x+iy)(x+iy) \\
&= x^2 + 2iyx + i^2y^2 \\
&= x^2 + 2iyx - y^2
\end{aligned}
$$

Julia Sets work with iterative equations, for example $F(z) = z^2$. $Z_0 = x_0 + iy_0$ with $|z| < 1$. Iterating this equation moves it closer to zero (see appendix). Thus, as long as $0<|z_0|<1$ then F(z) tends towards zero and consequently can be considered stable. If $|z_0| > 1$ then it follows that F(z) tends towards infinity and is also stable. This being the case, if $z_0 <> 1$ then it is stable.

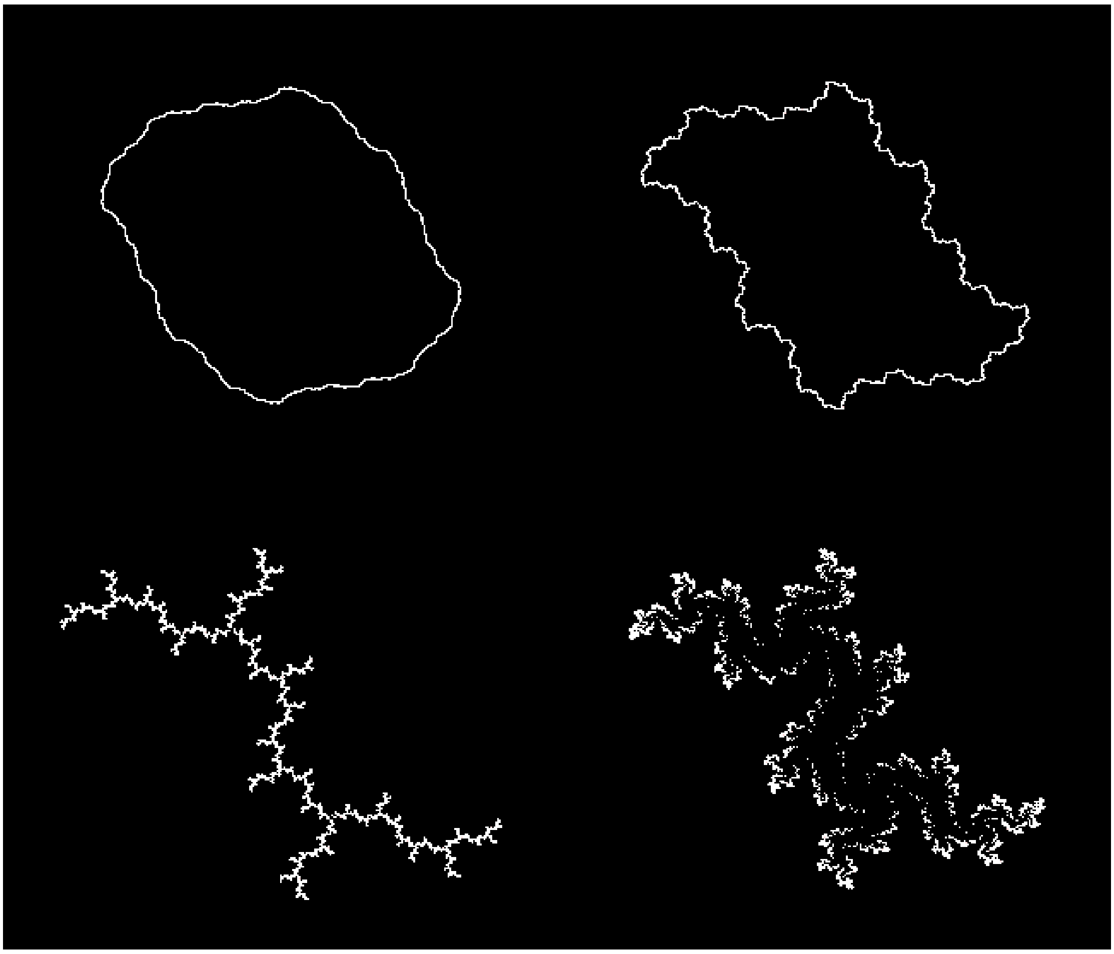
Julia Sets ($z = z^2 + c$) are defined by the constant c and consist of all points that do not tend to infinity or 0.

Julia Sets posses very similar properties to simple fractals such as the Koch curve, showing infinite complexity and self-similarity on differing scales.

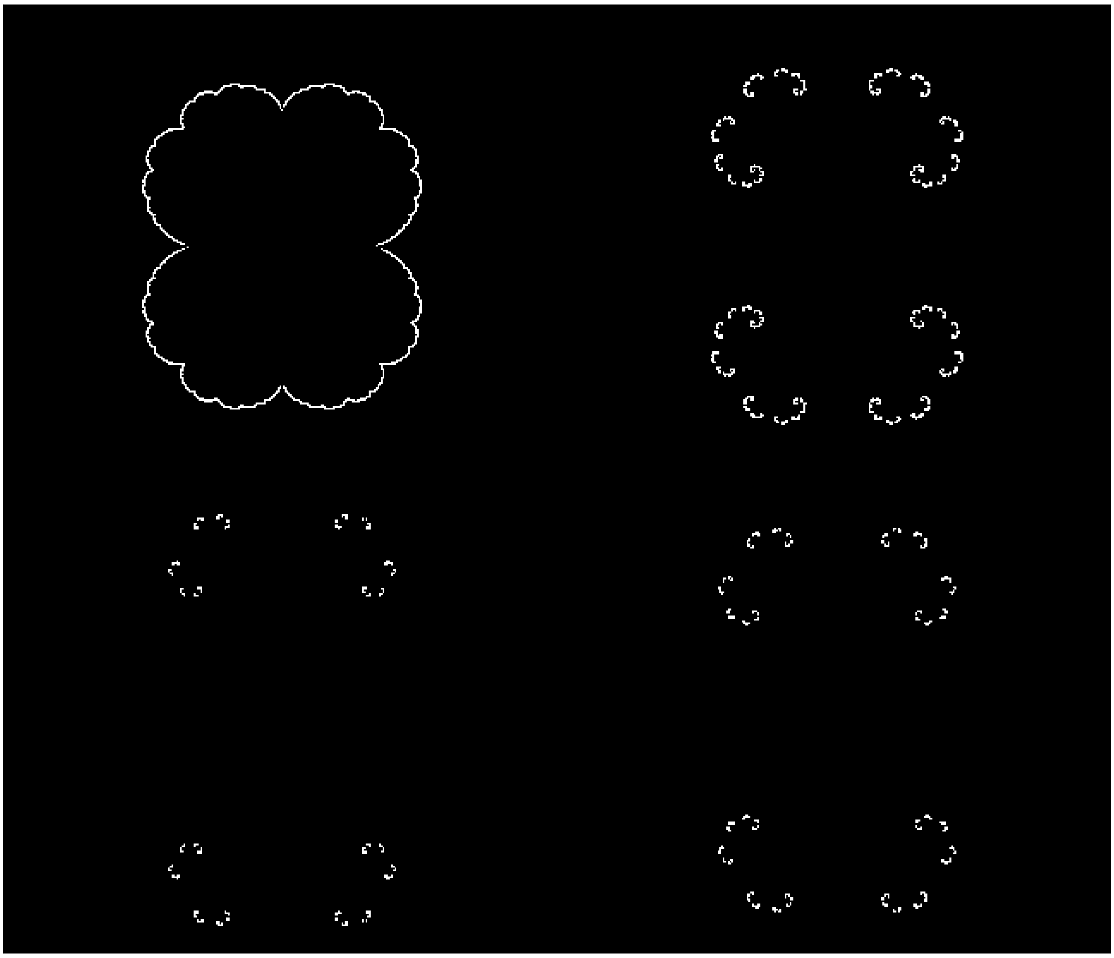
Through experimentation with my own models of Julia Sets, I have found that altering the real and imaginary parts of z have differing impacts on the form of the image, as described below.



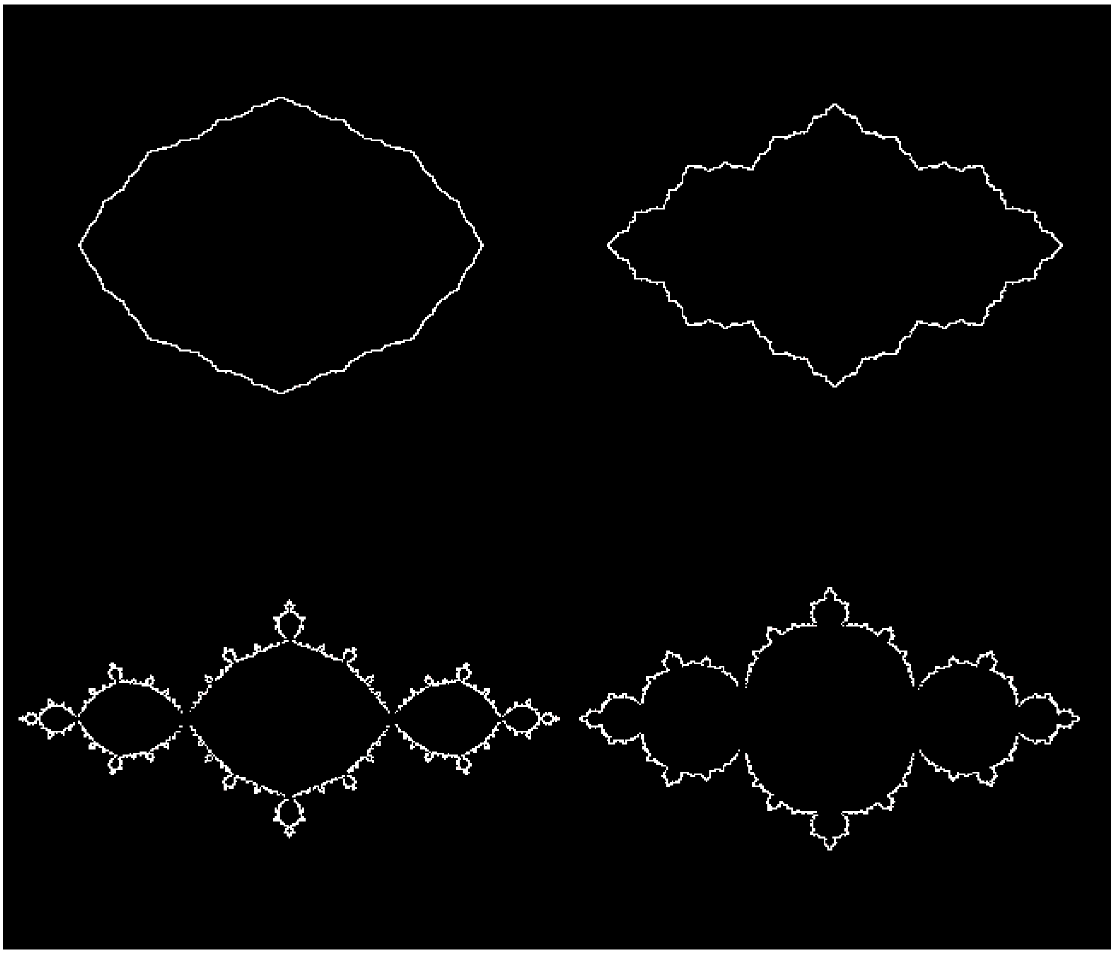
As soon as z is increased above 0 by even the smallest margin in either the real or imaginary part, the resulting image is of a near perfect circle. Hence, it is the starting image for the Julia Sets sequences displayed on the following pages. They are achieved by plotting a series of Julia Sets, increasing (or decreasing) either the x or the y value by a constant margin each time. For convenience, I refer to these as 'sequential Julia Sets' from here on.



These images show the y value, or the imaginary part of the complex number, being increased (clockwise from top left) as follows: 0.25, 0.5, 0.75, 1. Decreasing the y value (ie –0.25, -0.5 etc) simply mirrors each image.

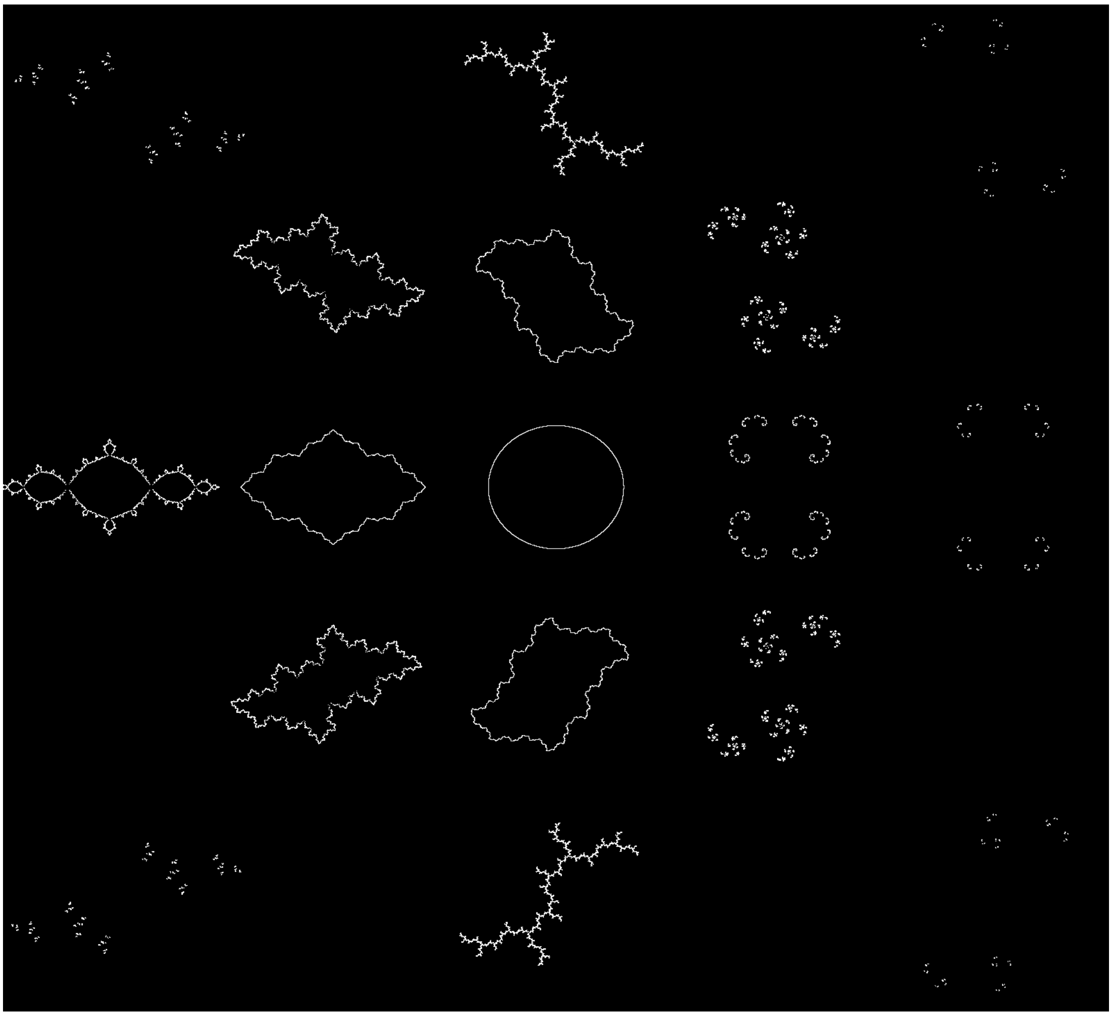


These images show the x value, or the real part of the complex number, being increased (clockwise from top left) as follows: 0.25, 0.5, 0.75, 1. Unlike with the y value, decreasing the x value results in a new set of images, shown below.

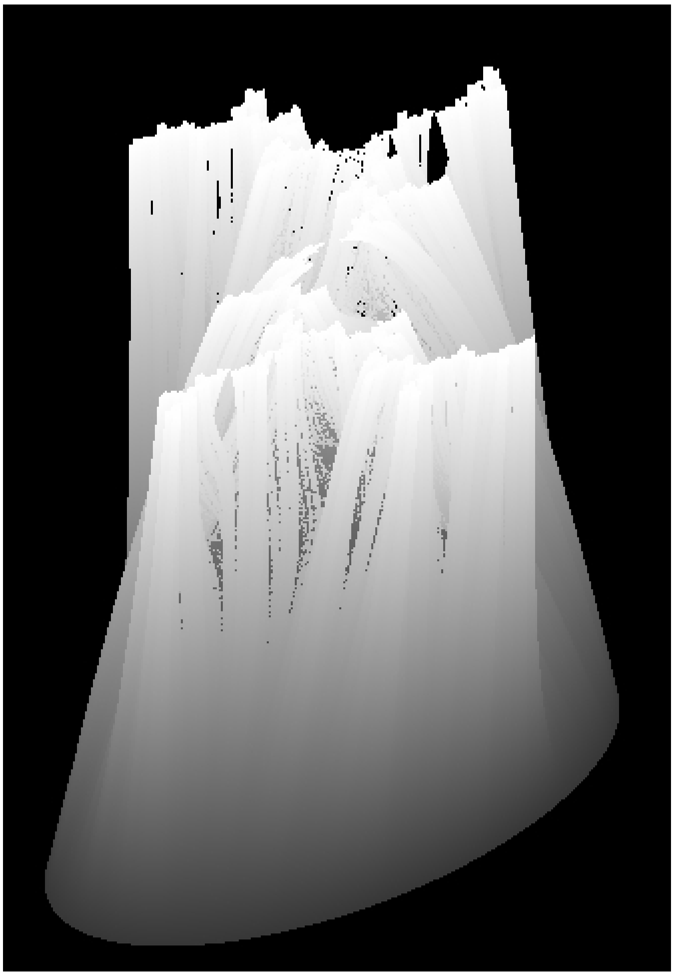


These images show the x value being decreased (clockwise from top left) as follows:

-0.25, -0.5, -0.75, -1.



This image shows Julia Sets respective to their place in the complex number plane. The circular Julia Set in the centre shows x = 0.00001, y = 0. Extending out from it are Julia Sets at intervals of 0.5. Hence, the Set in the bottom left had corner shows x = -1,y = -1. It clearly shows the transformation of sequential Julia Sets and the effects of the real and imaginary parts on the form of the image. The sets at 45 degrees to the x, y axis can be seen as a combination of their counterparts on the axis. For example, the set at 0.5, 0.5 is a combination of the sets at 0, 0.5 and 0.5, 0.

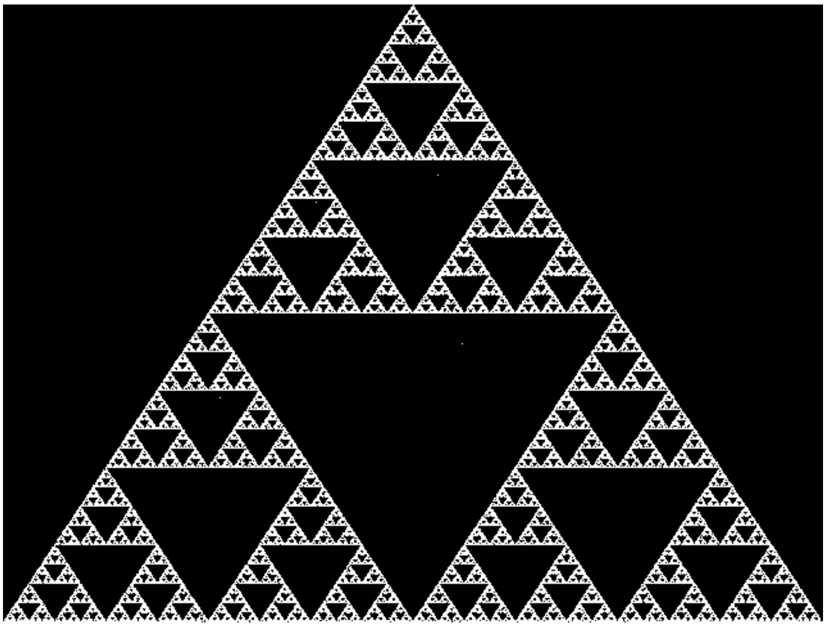


The above image shows 3-dimensional plotting of a sequential Julia Set. Each set is plotted on top of the previous, and hence, the method is able to show an entire sequential Julia Set simultaneously. My inspiration for this technique was drawn from Robert May (see section 4.5), who shed new light on bifurcations using a similar procedure. This set starts at 0,0 with the y value being increased by 0.001 for each of the 50,000 iterations.

Fractals can be used to create stunning visual landscapes with startling realism. Mandelbrot called it the geometry of nature for no insignificant reason. When looking at the world, Mandelbrot realised that much of the world around us cannot be explained by the traditional Euclidean geometry of spheres and cubes. As he said,

> *"Clouds are not spheres, mountains are not cones, coastlines are not circles and bark is not smooth, nor does lightning travel in a straight line."* [28]

Fractals, unlike Euclidean geometry, are able to mimic the "real" structures of the earth. For example, the earth, from a great distance, looks like a perfect sphere. As we get closer, the mountain ranges and other irregularities of the land become evident. From a distance, these mountains appear smooth. Looking in more detail, it is clear that the mountain itself has smaller irregularities, as if the mountain is composed of a series of miniature mountains. There are several geometrical examples of this, one of the most interesting is the Sierpinski triangle.



Each successively small white triangle is an exact replica of the overall triangle. Magnified sufficiently, any triangle will look identical to the original. Therefore, the triangle is infinitely complex, within infinite black space (black triangles) within the boundary of the white triangle. This prime example of complexity and infinity within a finite space is highly ordered but it is randomly generated using a few simple rules. The programme that created the above image places three points on the screen, one in each of the bottom corners (1 and 2) and one at the top in the centre (3). Then, the programme puts a random point (P_1) on the screen and randomly selects between one of the three original points. If 2 is selected, point P_2 is placed half way between P_1 and 2. This process continues ad infinitum and the image appears with increasing detail.

The self-similarity of fractals has also been noted by scientists in areas seemingly unrelated to fractal geometry, giving another good indication of their relevance to describing the natural word. Respected mining geologist Guy Lewington of Eagle Mining explains how in his days as a field geologist, he observed fractals of a kind in rock formations and riverbeds. Looking at a satellite photograph of a creek bed, he noticed mushroom-shaped rock formations about 10 kilometres in size. He then went into the creek bed and sighted an outcrop of the same shape. Taking a sample of rock from the area, he looked at it in thin section under a microscope and found, once again, the same formation.

As well as being able to graphically represent forms in nature, fractals have various practical applications. The film, *Star Trek II: The Wrath of Khan* by Lucasfilm uses several fractal-generated landscapes. These, apparently, were so realistic that many people did not even notice that computer graphics had been used. Later, Digital Productions used fractal landscapes in the film, *The Last Starfighter*.

Fractals have also been used to describe many natural phenomena including the structure of Saturn's rings.

5.3 Meteorology

During the 1950s, meteorologists had great hopes for weather forecasting under the banner of Newtonianism. Based upon the Laplacian idea that, with knowledge of the present, the future can be predicted, meteorologists were working with the view that through the use of new, powerful computers, weather prediction would become a simple matter of course. It was understood that a completely accurate knowledge of the weather is practically impossible, but small inaccuracies were viewed as being unimportant.

> *"The basic idea of Western science is that you don't have to take into account the falling of a leaf on some planet in another galaxy when you're trying to account for the motion of a billiard ball on a pool table on earth. Very small influences can be neglected. There's a convergence in the way things work, and arbitrarily small influences don't blow up to have arbitrarily large effects."* [29]

In other words, approximately accurate data gives an approximately accurate result. In the quoted example above, the effect of a falling leaf has such a small influence that the result on the billiard ball will be equally small – negligible.

Meteorologist Edward Lorenz (see section 4.4) found otherwise when examining the results of a weather simulating computer. The machine printed out a number that enabled Lorenz to interpret the weather his computer was producing. During the winter of 1961, Lorenz, wanting to examine a pattern again, re-entered the data from a previous print-out. As time progressed, the results came as a shock, not only to Lorenz, but also to science in general. The new print-out did not show an exact copy of the original. It started off in almost identical fashion but became more and more unrecognisable as time progressed.

It was not until Lorenz had examined his machine extensively for faults that he realised what was causing the discrepancy: whereas the computer calculated data to six decimal places, the print-out was only accurate to three. The inaccuracy of the starting value was only 1/1000, but through many calculations this small inaccuracy was being amplified over and over through a positive feedback loop.

The unpredictability Lorenz experienced with weather forecasting may be, at first, difficult to understand. After all, other similar predictions can be made for things such as tides and the orbits of planets. These predictions are so accurate that we often forget that they are predictions. What Lorenz points out is that these, in fact, are not so accurate as might be expected, although it is less obvious. It is barely noticeable if a comet that has been expected for nearly a hundred years arrives half an hour late.

Furthermore, long-term predictions can often be easier than those in the short-term. It is not hard to predict, for example, that next winter will be colder than this summer. To use one of Lorenz' examples,

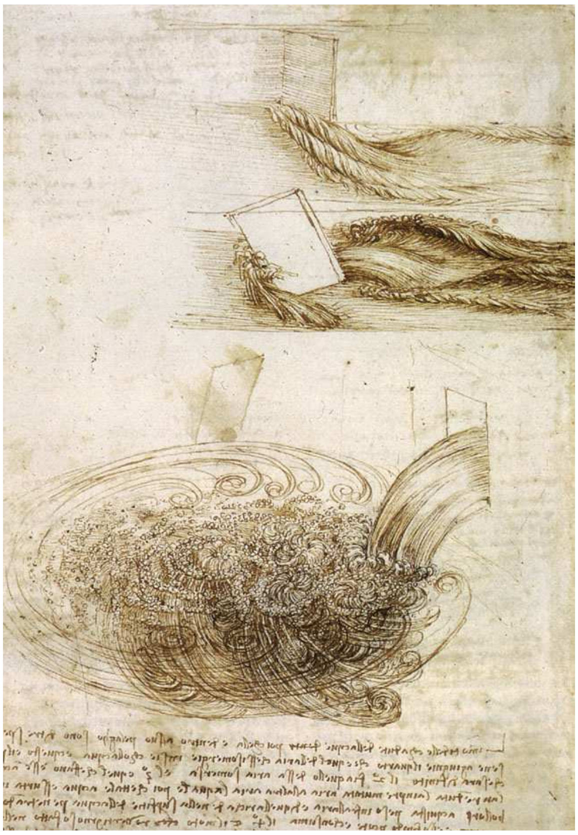
> *"We might have trouble forecasting the temperature of [a cup of] coffee one minute in advance, but we should have little difficulty in forecasting it an hour ahead."* [30]

Many scientists of the day had trouble with Lorenz' ideas at first. Seeing as just the flap of a butterfly's wings can change the weather, they saw the possibility for weather control. It would be easy, they thought, to manipulate the weather into a desirable position from which it would be easy to predict. It would, however, be like shuffling an already well-shuffled deck of cards. It is impossible to know what the result of either will be.

5.4 Fluid Dynamics

The Lorenz approach to chaos was through the field of fluid dynamics. This has been one of the most controversial areas to which chaos has been applied. Many fluid dynamicists rejected Lorenz' theories, because, like pendulums, they were thought to be well understood. In practical terms, fluid dynamics was well documented and was no longer considered part of physics by some, but a part of engineering. However, the change from smooth flow to turbulent flow was not understood. For many, fluid dynamics seemed unexplainable.

5.4.1 Leonardo Da Vinci



The great Renaissance artist, Leonardo Da Vinci, was one of the first researchers in the field of fluid dynamics and turbulence. He used his artistic abilities to document fluids in turbulent motion, the results of which are shown opposite.

In his work, he uncovered a process similar to the bifurcations discussed in section 5.1.1. Eddies fragment into smaller and smaller eddies, resulting in turbulence. This is known as the 'period doubling route to chaos.' Although turbulence looks very similar to other forms of bifurcatory behaviour, it is unclear as to whether the 'windows' of order previously described can be seen.

Despite the avid interest of Leonardo and others, such as Lord Kelvin, turbulence remained a backwater field of study until recently, when chaos shed new light on the subject.

5.4.2 Turbulence

Turbulence is something that science has always had difficulty explaining, since it is very difficult to model. Water travelling down a pipe has no outside influences that could possibly induce turbulent motion, and yet, if the volume of water is high enough, turbulence appears, seemingly from nowhere.

One of the simplest ways to look at fluids that develop turbulent motion is a Taylor-Couette system, first studied at Cambridge by Geoffry Ingram Taylor during the 1920s. This, basically, consists of two cylinders, one inside the other. The outer cylinder remains stationary while the inner cylinder turns to create movement of the fluid in between.

The equations that explain fluid motion are called Navier-Stokes equations after Claude Navier (1785-1836) and George Stokes (1819-1903) who developed them independently. Being based on Newton's laws of motion, they are deterministic. As we have seen, this does not necessarily mean that it is a simply matter to make predictions. The Navier-Stokes equations are non-linear, meaning that there is a large potential for chaotic behaviour in the system. This has been demonstrated in section 5.1.2 using the simple mathematical population model $x_{next} = x * \lambda * (1 - x)$.

When the water flow is slow, the Taylor-Couette System has few surprises. The flow is mainly in concentric circles around the axis of the cylinders. However, most of the fluid movement in nature is chaotic and as "*…laminar flow is not usually found in nature, …it does not have much practical value.*" [31]

As the speed of the inner cylinder increases, a secondary motion suddenly appears, superimposed upon the first. This motion has been described as "stacked Swiss rolls" that run horizontally to the vertical Taylor-Couette system. Increasing the speed yet further adds another, separate motion. The faster the system is run, the smaller the increase in speed needed to induce a new motion. Because of this, it would not be long before all possible motions were in action and, according to a Lev Landau theorem put forward in 1944, this was turbulence.

Since then, however, this idea has been challenged. Several scientists have been working with theories more in line with the mathematical model mentioned above. One researcher, Gerd Pfister of the University of Kiel, uncovered a period-doubling in fluid motion using a miniature (and therefore simpler) version of the Taylor-Couette system. According to Tom Mullin of the Clarendon Laboratory at Oxford, visual representations of this procedure are qualitatively the same as that of the simple differential equations described in section 5.1.1. The parallels between this system and Robert May's bifurcations are immediately obvious. Analogies can be extended further, Mullin says, to situations such as chemical oscillators and lasers. This is another of the major features of chaos. There is a universality that crosses the borders of conventional disciplines.

Although turbulence is far from being understood, *"mathematical ideas of chaos may have found a chink in the armour"*[32] of a mystery that lead British physicist, Horace Lamb, to say:

> *"I am an old man now, and when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really rather optimistic."* [33]

6 Computer Models of Chaos

Much of chaos theory would not be possible without the number crunching power of the computer. In order to view May's bifurcation diagram to the standard shown in this report requires 5,000 calculations for each point, and to create the animated Julia set image required 5,050,000 calculations. While computers can achieve this in a matter of minutes with unerring accuracy, consider the amount of time and concentration it would take for a human to achieve the same result. Even if someone went to the trouble of doing the calculations, there is a large possibility that some inaccuracy would occur and hence, the entire exercise would be worthless.

Computers, therefore, have played and continue to play a large part in the development of chaos theory, from its discovery by Lorenz and Mandelbrot, to more recent work in modelling insulin secreting cell cooperation and stomach tissue voltage patterns.

The following programmes provide a few basic demonstrations of chaotic behaviour and fractal imagery.

6.1 Mathematics and Populations

To be updated tomorrow – see appendix for code.

6.2 Julia Sets

Julia Sets. Plots a single Julia set. Allows user to enter value for both real and imagery parts of complex number, and iterations. See appendix.

3-dimensional sequential Julia Sets. Plots Julia Sets on a 3-axis complex number plane. Allows user to define increments, value to increment (x or y), no of iterations, contour lines, and initial and ending values. See appendix for code.

7 Conclusions

"No doubt… we live on a planet dominated by chaotic principles."[34]

"Chaos has become not just theory but also method, not just a canon of beliefs [mythology?] but also a way of doing science."[35]

Common to many mythologies throughout the world is the understanding of chaos as a state from which the ordered world developed. Often it takes the metaphor of water, and many cosmogonies that do not actually name chaos as such, acknowledge the concept through the image of water.

The mythologies of Mesopotamia, Egypt, and China, see chaos as an entity to be controlled or suppressed, and in the case of the latter two, it was seen as a force that had the potential to revert order back to chaos. They perceived a constant struggle between chaos and order, that is, the ability for one to become the other.

Lighting, one of nature's best exponents of fractal structure, plays a part in the emergence of order in Chinese and Indian mythology.

The Greeks represented a transition from old mythologies to modern scientific method. In this process, the ideas of chaos were lost through the process of reductionism, culminating in Newton's all-encompassing theories during the seventeenth century. Although there were still indications that chaos may exist within

Newtonian determinism, the technology of the day was insufficient to bring any conclusive evidence.

With the introduction of the computer midway through this century, science had the means to expose chaos, and discover the patterns within it. Since then, chaos has developed into one of science’s best models of the natural world. Fractals have provided the geometric base for chaos theory, with primitive Julia Sets supplying a base for stunning fractal images that are able to mimic plant life and landscapes with incredible realism.

Chaos in mythology and cosmogony possess universality through the constant themes of primordial timelessness and infinity. Universality is also one of the strongest features of chaos theory, where themes cross the borders of scientific disciplines. The concept of infinity is omnipresent in both myth and modern theory, fantastic ending sentence.

Appendices

Appendix 1: Feedback Loops

Feedback loops, both positive and negative, are common in our everyday lives. A negative feedback involves one variable affecting a second variable, which in turn affects the first. A common example of this is an oven thermostat. When the oven reaches a preset temperature, the thermostat turns the oven off, causing the oven to cool. When the oven cools beyond a certain point, the thermostat turns the oven on again and the temperature rises. In this system, the thermostat and temperature influence each other to maintain a stable temperature. An example of a positive feedback loop is the feedback noise made when a microphone is placed too close to a speaker. The sound of the speaker is picked up by the microphone, amplified, and re-emitted out through the speaker, where it is, once again, fed back into the microphone.

> *"The chaotic sound is the result of an amplifying process in which the output of one stage becomes the input of another."* [36]

The ideas of positive feedback and sensitivity on initial conditions are not unique to modern science. The writer J.B. Priestly examined the idea in a play called "Dangerous Corner", which has two entirely different outcomes, both wholly dependant on whether a simply question regarding a cigarette box is asked. The notion also has a place in folklore:

For want of a nail, the shoe was lost;

For want of a shoe, the horse was lost;

For want of a horse, the rider was lost;

For want of a rider, the battle was lost;

For want of a battle, the kingdom was lost!

And all for the sake of a horse shoe nail.

Appendix 2: Iterations.

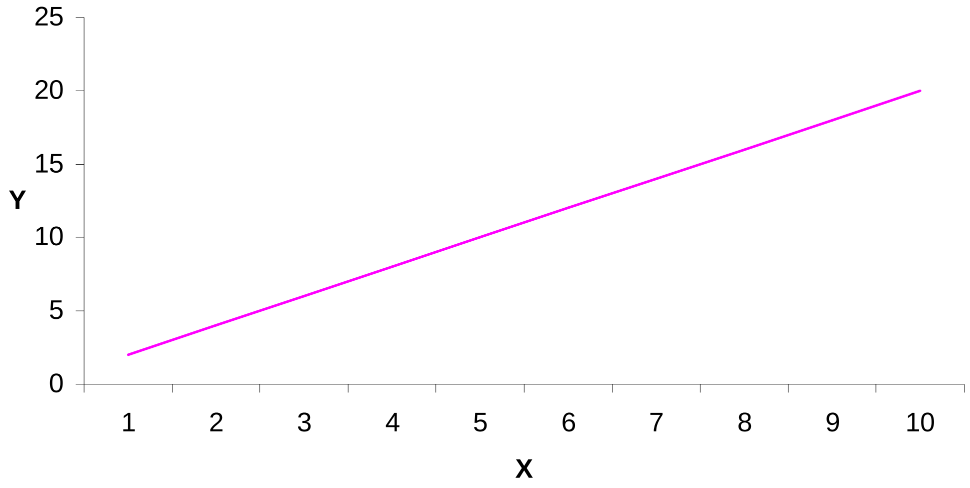
Iterations refer to the number of times an equation run. An iterative equation is one were the answer is re-entered into the equation to gain the next result. If an equation is run ten times then it has been through ten iterations.

Example: $x_{next} = x + 3, x_0 = 0$

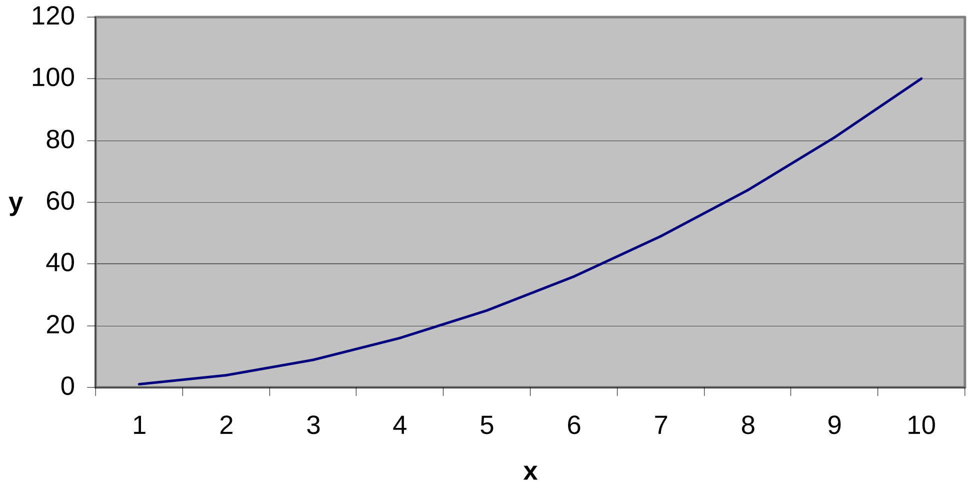
Iterations	Result
1	$x_{next} = 0 + 3$ therefore x = 3
2	$x_{next} = 3 + 3$ therefore x = 6
3	$x_{next} = 6 + 3$ therefore x = 9

Appendix 3: Linearity

A linear equation is one that, when graphed, form a straight line, for example, x = 2y. The graph of this is as follows:



Non-linearity describes an equation that forms a graph that is not a straight line. An example of this type of equation is as follows: $x^2 = y$



Linear equations are much easier to work with than non-linear. With a linear equation, a formula can be found to quickly solve the equation for the nth iteration. In other words, they are predictable and consequently, the equation doesn't have to be run to find a certain result. Example: $x_{next} = x + 3$. To compute this equation to the nth iteration, the formula would be as follows: $x = n * 3$.

The majority of non-linear equations are non-predictable.

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